

Math 381 Complex Variables and Transforms

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due in class 13:35, Jan 18

Homework 1

Exercise 1.1. Find the real part $\operatorname{Re} z$ and the imaginary part $\operatorname{Im} z$ of the following complex numbers

1. $z := (3 - i)(5 + 2i)$,

3. $z := \exp 2\pi i/6$.

2. $z := \frac{1+i}{1-i}$,

Exercise 1.2. Find all possible solutions $z \in \mathbb{C}$ to the equations

1. $z^6 = 2$,

2. $z^2 - 2z + 1 = -1$.

Exercise 1.3. Sketch the following subsets in the complex plane \mathbb{C} :

1. $\{z \in \mathbb{C} : |z| < 1\}$,

3. $\{z \in \mathbb{C} : (\operatorname{Re} z)^2 < 1\}$,

2. $\{z \in \mathbb{C} : |z - 1| \leq 1\}$,

4. $\{z \in \mathbb{C} : 1 \leq |z - 1| \leq 2\}$.

Exercise 1.4. For two real numbers $a, b \in \mathbb{R}$ derive the equation

$$\cos(a + b) = \cos a \cos b - \sin a \sin b.$$

Hint: Consider $\exp((a + b)i) = \exp(ai) \exp(bi)$, which will be derived in class.

Exercise 1.5. For a complex number $q \neq 1$ show

$$\frac{1 - q^{n+1}}{1 - q} = \sum_{k=0}^n q^k.$$