Math 381 Complex Variables and Transforms

Instructor: Matthias Nagel — due in class 13:35, Mar 7

Homework 4

The function $\arctan\colon \mathbb{C} \setminus \{\pm it : t > 1\} \to \mathbb{C}$ is defined by

$$\arctan(z) := \int_0^z \frac{1}{1+u^2} du.$$

Recall that $\lim_{k\to\infty} \left(1 + \frac{1}{k}\right)^k = e$ and $\binom{\sigma}{0} := 1$ and for all other natural numbers n:

$$\binom{\sigma}{n} := \frac{\sigma(\sigma-1)\cdots(\sigma-n+1)}{n!}$$

Exercise 4.1. Invert the function $\cos z$ near the point $\frac{\pi}{3}$ and express it in terms of branches of the logarithm and the square root function.

Exercise 4.2. Calculate the radius of convergence of the following series:

1. $\sum_k k^2 z^k$
2. $\sum_k \frac{k^k}{k!} z^k$
3. $\sum_k \exp(k) z^{2k}$
4. $\sum_k \binom{\sigma}{k} z^k$ for a complex number σ.

Exercise 4.3. Expand the following function $f(z)$ into a power series $\sum_k a_k (z - z_0)^k$ around the the given point z_0:

1. $f(z) := \frac{z^3}{z^2+1}$ with $z_0 = -1$.
2. $f(z) := \arctan(z)$ with $z_0 = 0$.
3. $f(z) := \cos z \cdot \sin z$ with $z_0 = 0$.
4. $f(z) := \frac{\cosh z}{1-z}$ with $z_0 = 0$.

Exercise 4.4. Determine $B_0, \dots, B_3$ of the expansion

$$\frac{z}{e^z - 1} = \sum_k B_k z^k.$$