

Voluntary Homework 6

Exercise 6.1. Let X, Y be path-connected topological space.

- (1) Compute the homology of the disjoint union $X \sqcup Y$.
- (2) Compute the homology of the wedge $X \vee Y$.

Exercise 6.2. Compute the homology of the complement $D^n \setminus \{x_1, x_2\}$ of two distinct points $x_1, x_2 \in \text{Int } D^n$ for $n \geq 2$.

Exercise 6.3. Compute the homology of the suspension ΣX in terms of the homology groups of X .

Exercise 6.4. Compute the homology of the 2-torus $S^1 \times S^1$.

Exercise 6.5. Let $f: S^1 \rightarrow S^1$ be map $z \mapsto z^2$. Compute the homology groups of $X = S^1 \cup_f D^2$ with \mathbb{Z} and $\mathbb{Z}/2$ -coefficients.

Exercise 6.6. Consider the pair of pants surface Σ , depicted below. Its boundary consists of three circles $\partial\Sigma \cong S^1 \sqcup S^1 \sqcup S^1$. Compute $H_2(\Sigma, \partial\Sigma; \mathbb{Z})$.

